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Inventory management under service level constraints with dynamic advanced order information

Heinrich Kuhn
Catholic University of Eichstätt-Ingolstadt, heinrich.kuhn@ku-eichstaett.de

Andreas Popp
Catholic University of Eichstätt-Ingolstadt, andreas.popp@ku-eichstaett.de

Advanced Order Information (AOI) occurs in a make-to-stock environment with stochastic demand when a customer issues his order before his desired delivery date. This extra time span offers the manufacturer the chance to reduce his safety stocks. When several customers issue their orders for a particular delivery date at a different point in time, Dynamic Advanced Order Information (DAOI) occurs. Thus, for each planning cycle the planner has different information about the actual level of demand for the upcoming periods. The paper discusses the impact of Dynamic Advanced Order Information on an inventory system following an (r, S_t) inventory policy. Conditional distributions are used to model Dynamic Advanced Order Information.

Key words: Inventory management; Advanced Order Information; (r, S_t) inventory policy

1. Introduction and literature review

The paper considers the advantage of Advanced Order Information (AOI) in a dynamic business-to-business (B2B) environment of an inventory system. In a business-to-business (B2B) environment a supplier faces generally a different demand pattern than in a business-to-customer (B2C) environment. In a B2C environment the retailer is concerned with many customers who usually order or buy a small amount of a product that should be delivered immediately. In a B2B environment, however, the industrial partners order larger batches of each product. Thus, the industrial supplier is usually in contact with a limited number of industrial customers that order large quantities of a product for a specific point of time (scheduled date of delivery). However, the delivery times (lead times) may be shorter than the production time (throughput time). In this case, the industrial supplier has to have a certain amount of inventory on hand or he has to release production orders to the manufacturing system in advance, i.e. before customer orders arrive. In general, the following structural condition exists: the earlier a customer requests its order (i.e. long delivery times), the less inventory level (safety stock) is necessary to achieve a certain service level.

The pre-given delivery time for a customer order is called Advanced Order Information (AOI). In literature AOI is a field of research, with some quite different approaches. Most of them consider a fixed customer lead time and a single (aggregated) product. Wijngaard (2002) analyzes a system with AOI in which a given utilization rate is to be realized with as little inventory as possible for the lost-sales and the backorder case. Gallego and Özer (2001) study state dependent (s, S) and base stock inventory policies to find optimal policies for inventory problems with AOI. Karaesmen et al. (2002) use queuing theory to analyze a make-to-stock queue with a fixed visibility horizon which is caused by AOI. Wijngaard and Karaesmen (2007) consider a reward driven make-to-stock model with fixed customer lead time to quantify the benefit of AOI. Gilbert and Ballou (1999) conduct a case study on an inventory model with AOI to determine price discount incentives for customers.

This paper focuses on an adapting dynamic change of AOI into a service level driven inventory policy for a single product.

2. Problem description

We assume inventory is planned according to a (r, S_T) inventory policy, where T is the considered planning period. Thus, the planner reviews the inventory in fixed, discrete time intervals (e.g. each week) and restocks the inventory on hand to a level S_T. Each planning cycle, the parameter S_T is newly determined. The assumed policy equals a lot-for-lot policy, where the planner calculates and dispatches a new production lot in size P_T at the beginning of each planning period. The demand in each period is stochastic and follows a known probability distribution; unsatisfied demand is back-ordered. The aim of the supplier is to minimize its inventory on hand while a pre-given service level (β_0 or γ_0) is fulfilled in each period. The underlying assumptions lead to the following optimization models:

$$\begin{array}{lllll} \min & P_T & & \min & P_T \\ \text{s.t.} & \beta(P_T) \geq \beta_0 & \text{respectively} & \text{s.t.} & \gamma(P_T) \geq \gamma_0 \\ & P_T \geq 0 & & & P_T \geq 0 \end{array}$$

Note, that the model only optimizes the lot-size for one period. Hence, the service level will not be averaged over the periods when planning several periods in advance.

To gain a progressive insight into the potential of AOI we assume that the demand for a specific period is issued by the customers in different periods. Hence, the demand for period T is made up of different customer orders which are placed at different points in time. Given a fixed production lead time L_P the planner has to dispatch the lot for period T by no later than period $t := T - L_P$. As shown in Figure 1 by then, a share of the orders is already issued in period t and therefore certain. Every order which arrives with less customer lead time than L_P periods is uncertain in period t and therefore has to be forecasted, hence additional safety stock is needed to keep the pre-given service level. The problem can be interpreted as a service level driven Wagner-Within-Problem without setup and with positive production lead time. A service level driven, stochastic variant of the Wagner-Within-Problem is analyzed by Tempelmeier and Herpers (2009).

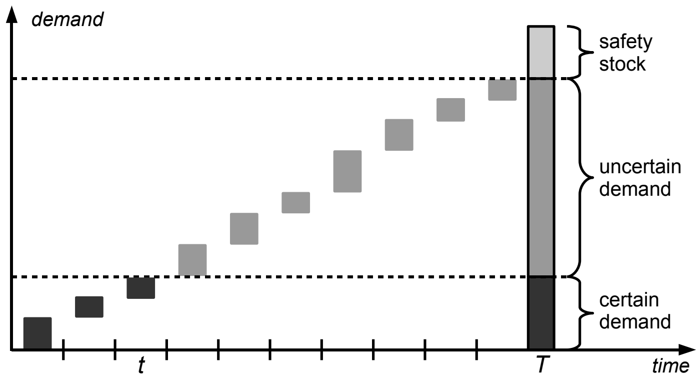

Figure 1 Dynamic Advanced Order Information

As illustrated in Figure 2 this information gets more precise with each period. At the first view it seems as if the information of orders within the production lead time is wasted, since the lot for period t has already been dispatched when orders in period $t+1$ arrive for period T. However, the information may still be used for planning period $T+1$. The information that the demand in period T was over- or underestimated gives additional information about the safety stock having been dimensioned to large or to small.

If, for example, a very large, unforeseen order is issued in period $t+1$ for period T, which implies that the safety stock for period T will be insufficient, the planner may increase the dispatch quantity for period $T+1$, so that the back order only lasts for one period. Without the consideration of the new information, the demand distribution D_T is assumed to be static and the unlikely high demand would first be noted in period T.

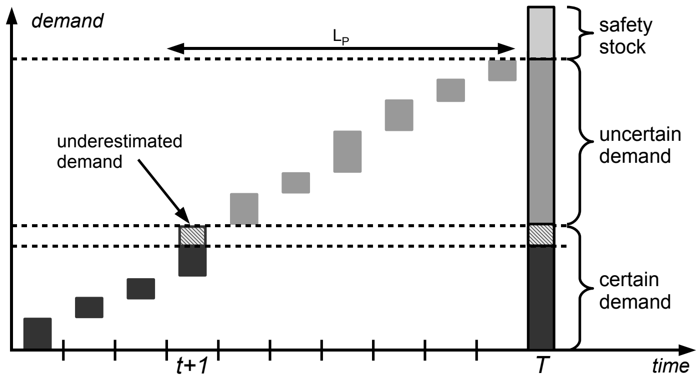

Figure 2 Gain of information about actual demand in period T

3. Mathematical theory for problem analysis

3.1. Calculating the service levels

To make use of advanced order information we will first analyze the service level functions $\beta(P_T)$ and $\gamma(P_T)$. The basic calculus is similar to Fischer et al. (2007), however, in this paper we will use more rigor mathematics to develop a closed form expression. In the first instance we consider the random variable I_T^{End} which denotes the inventory at the end of period T. When planning in period t the inventory level I_t^{End} is certain and is therefore denoted with i_t^{End}. The variation of the stock level between periods t and T is caused by the variability of the demand and production outcome in the intermediate periods, so it holds:

$$I_T^{End} = i_t^{End} + \sum_{j=t+1}^{T} P_j - D_j \tag{1}$$

The demand volumes D_j for each period $j = t+1, t+2, ..., T$ are random variables. The production lot sizes $P_{t+1}, \ldots, P_{T-1}$ have been already fixed in the previous periods and P_T is the considered decision variable. Therefore, we convert equation (1) by summing up all non-decision variables to variable I_T^*, which denotes the inventory at the end of period T excluding the production lot P_T. The distribution of I_T^* is given by the following convolution:

$$I_T^{End} = \underbrace{i_t^{End} + \sum_{j=t+1}^{T-1} P_j - D_j - D_T}_{I_T^*} + P_T = I_T^* + P_T \tag{2}$$

The service levels are defined as in Tempelmeier (2006) pp 26–27:

$$\beta = 1 - \frac{E(B_T^{End}) - E(B_T^{Beg})}{D_T} \qquad \text{respectively} \qquad \gamma = 1 - \frac{E(B_T^{End})}{D_T}$$

The specified expected values of the backorders at the end and the beginning of period, T, $E(B_T^{End})$ and $E(B_T^{Beg})$, are functions of the decision variable P_T. This is shown by the following calculus. The backorders are the negative branch of the inventory level I_T^{End}. This means, that positive inventory levels are treated as backorder volume 0 and negative inventory levels are treated as backorders only with a negative algebraic sign, to convert the backorders to positive levels. So, for calculating the expected value of the backorders at the end of period T it holds:

$$E(B_T^{End}) = \underbrace{\int_0^\infty 0 \, dP^{I_T^{End}}(x)}_{=0} - \int_{-\infty}^0 x \, dP^{I_T^{End}}(x) \overset{(2)}{=} -\int_{-\infty}^0 x \, dP^{I_T^* + P_T}(x) = -\int_{-\infty}^{-P_T} x + P_T \, dP^{I_T^*}(x) \tag{3}$$

The calculation is similar for $E(B_T^{Beg})$, the only difference is that the demand D_T is not subtracted. Thus, instead of I_T^*, the demand values and lot sizes for the periods $t+1, \ldots, T-1$ are combined to I_{T-1}^{End}. The closed form expressions for both functions results as follows:

$$\beta(P_T) = 1 - \frac{-\int_{-\infty}^{-P_T} x + P_T \; dP^{I_T^*}(x) + \int_{-\infty}^{-P_T} x + P_T \; dP^{I_{T-1}^{End}}(x)}{E(D_T)} \tag{4}$$

and

$$\gamma(P_T) = 1 - \frac{-\int_{-\infty}^{-P_T} x + P_T \; dP^{I_T^*}(x)}{E(D_T)} \tag{5}$$

Note, that in all these calculations a deterministic production lead time is assumed. However, the model may easily be reformulated for finite discrete random lead times by using the total probability theorem. Thus, the closed form expression of the service levels, which is the only constraint, allows the quantification of the service levels. During the calculation we only have to sum up the results for all combinations of order lead times multiplied with their probabilities.

3.2. Using conditional distributions for implementing DAOI

The possibility to implement DAOI into the given model, lies in the distribution of the demand variables D_i. Generally, we assume the distribution is known. So, in practice one would try to fit a distribution to a given set of demand data and from then assume that the demand is distributed as in the result. This static point of view does not consider different customer lead times. Therefore, we split the demand into several periodical orders according to Chapter 2. The random Variable D_T^τ denotes the order amount for period T which is issued in period τ, d_T^0 denotes the orders already issued until period t.

$$D_T = d_T^0 + \sum_{\tau=t+1}^{T} D_T^\tau \tag{6}$$

When planning in period $t+1$ for period $T+1$ the random variable D_T^{t+1} is realized with value d_T^{t+1}. Thus, the distribution of the overall demand D_T for period T has to be adjusted for planning period $T+1$.

$$P^{D_T | D_T^{t+1} = d_T^{t+1}} = P^{d_T^0 + d_T^{t+1} + (D_T^{t+1} | D_T^{t+1} = d_T^{t+1}) + \ldots + (D_T^T | D_T^{t+1} = d_T^{t+1})} \tag{7}$$

For the easiest case of independent order volumes this formula is reduced to the sum of certain and random time-based orders.

$$P^{D_T | D_T^{t+1} = d_T^{t+1}} = P^{d_T^0 + d_T^{t+1} + D_T^{t+1} + \ldots + D_T^T} \tag{8}$$

Following this pattern we calculate the demand distribution again for each planning cycle, assuming suitable values for m.

$$P^{D_T|D_T^{t+1}=d_T^{t+1},\ldots,D_T^{t+m}=d_T^{t+m}} = P^{d_T^0+\sum_{\tau=t+1}^{t+m} d_T^\tau+\sum_{\tau=t+m+1}^{T} D_T^\tau} \tag{9}$$

For non-independent order volumes, D_T^τ has to be changed to $(D_T^\tau|D_T^{t+1}=d_T^{t+1},\ldots,D_T^{t+m}=d_T^{t+m})$.

4. Solution approach

Since the model consists of one single decision variable and one single restriction only, we may employ easy optimization techniques. An obvious fact, which may easily been proven, is that the service levels are monotonously dependent on the stock level respectively the lot size. Hence, a larger lot size will not worsen the service level. This monotonous dependency can be used to solve the model with nested interval, since we only need to find the minimal lot size which hits the minimum service level.

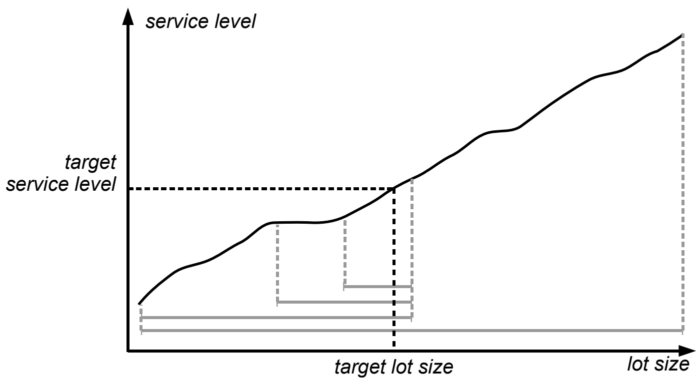

Figure 3 Method of nested intervals for a monotonous function

The procedure starts with a lower and an upper limit of the production quantity (search interval), where the target lot size is obviously included. Then the service level for the middle of the interval is computed. If this service level exceeds the target service level we use the lower half of the interval for the next iteration. If it undershoots the service level the upper one is used. The iterations are repeated until the interval length falls below a pre-given ε-value.

5. Numerical results

To gain a first idea about the advantage of using dynamic AOI we simulated the considered dynamic inventory policy with 3 data sets (A, B, and C). The results are compared with results neglecting the dynamic AOI. For easy convolution of the demand distribution, the order volumes D_T^τ are iid $N(\mu,\sigma)$-distributed, where μ is a random value between 1000 and 10000 an σ is computed out of a given coefficient of variance ($CV_A=0.15$, $CV_B=0.5$, $CV_C=1.0$). The production lead time is three periods, the target β-service level is 0.98 and the planning horizon is 30 periods. Orders are allowed to be issued until 10 periods before their delivery dates. For each data set 10 products with 10 replications (which equals 100 replications) are computed and the mean reach (relative inventory level) is calculated. The results are compared with a t-test. All computations are done with GNU R, the integration is done with the trapezoidal rule, since the distributions in data set A are to slender for the embedded integration function of GNU R.

H_0	p-value	95%-conf.-interv.	mean value
$\mu_{noAOI} \leq \mu_{AOI}$	$2,1 \cdot 10^{-7}$	$[0.00790627; \infty[$	0.0270834 / 0.0171791

Table 1 t-test results for dummy data set A

H_0	p-value	95%-conf.-interv.	mean value
$\mu_{noAOI} \leq \mu_{AOI}$	$5,4 \cdot 10^{-7}$	$[0.0386997; \infty[$	0.1871693 / 0.1364404

Table 2 t-test results for dummy data set B

H_0	p-value	95%-conf.-interv.	mean value
$\mu_{noAOI} \leq \mu_{AOI}$	$2,7 \cdot 10^{-5}$	$[0.07432596; \infty[$	0.4945362 / 0.3697614

Table 3 t-test results for dummy data set C

The results show that when using dynamic AOI a significant reduced inventory level will be achieved. However, the results did not show that the degree of the advantage depends on the demand variability. The formulated hypothesize regarding the demand variability was, that AOI might be more useful for higher demand variability, since in this case less advanced information is known. The results show indeed that the absolute differences are the largest in data set C. However, considering the mean values the relative differences of all test sets are about the same. Thus, the increase in the absolute differences might be caused by the increase of the overall inventory on hand. In general, a higher demand variability will not necessarily boost the application of AOI. However, it may be possible that the considered test cases were to similar in respect of the demand variability because of the variation diminishing effect and the high ratio of certain demand. Further researches have to be done with additional data sets.

References

Fischer, Lars, Sascha Herpers, Michael Manitz. 2007. Service-level oriented lot sizing under stochastic demand. J. Kalcsics, S. Nickel, eds., *Operations Research Proceedings 2007*. Springer, Berlin, 351–356.

Gallego, Guillermo, Özalp Özer. 2001. Integrating replenischment decisions with advance demand information. *Management Science* **47** 1344–1360.

Gilbert, Stephen M., Ronald H. Ballou. 1999. Supply chain benefits from advanced customer commitments. *Journal of Operations Management* **18**(1) 61–73.

Karaesmen, Fikri, John A. Buzacott, Yves Dallery. 2002. Integrating advance order information into make-to-stock production systems. *IIE Transactions* **34** 649–662.

Tempelmeier, H. 2006. *Inventory Management in Supply-Networks - Problems, Models, Solutions*. Books on Demand, Norderstedt.

Tempelmeier, H., S. Herpers. 2009. Dynamic uncapacitated lot sizing with random demand under a fill-rate constraint. Tech. rep., University of Cologne, Department of Supply Chain Management and Production, Cologne.

Wijngaard, J. 2002. The effect of foreknowledge of demand in case of a restricted capacity: The single-stage, single-product case with lost sales. *European Journal of Operational Research* **159** 95–109.

Wijngaard, Jacob, Fikri Karaesmen. 2007. Advance demand information and a restricted production capacity: on the optimality of order base-stock policies. *OR Spectrum* **29** 643–660.